Fatigue scatter model and non linear cumulative damage

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Pôle Méca seminar series

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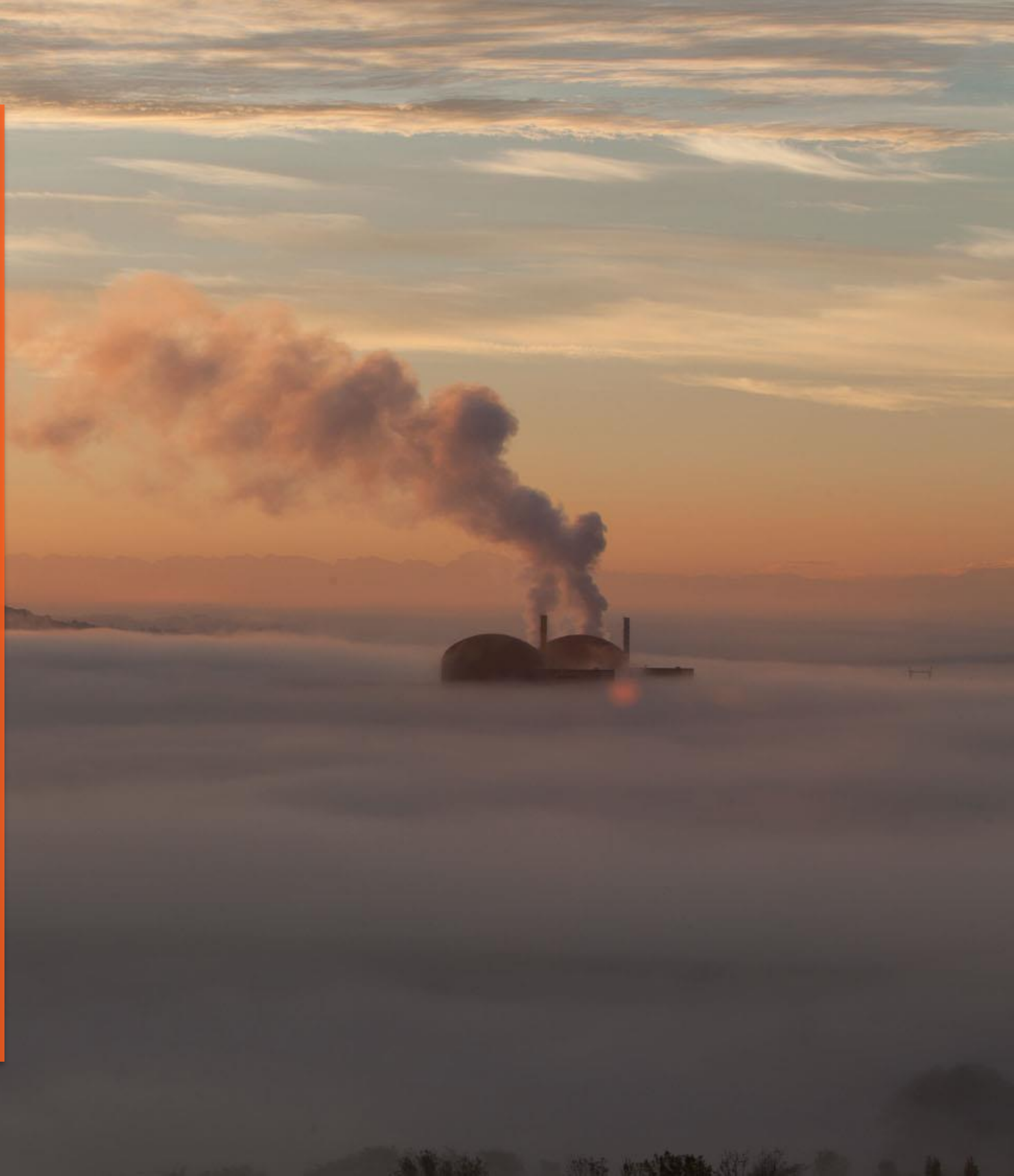


Context

- French codified nuclear methods (i.e., RCC-M) used to assess fatigue damage
  - **Deterministic** rules
  - Simplified methods → **Conservative assumptions**
  - Still submited to a lot of uncertainties

→ **How much is the fatigue criterion (Miner fatigue criterion $U < 1$) conservative ?**

- Probabilistic studies used to measure the codified criteria margins
  - Uncertainties in the methods modeled as probabilistic laws
    → **Requires a good knowledge of the uncertainties sources !**
  - Propagation of uncertainties in the fatigue models

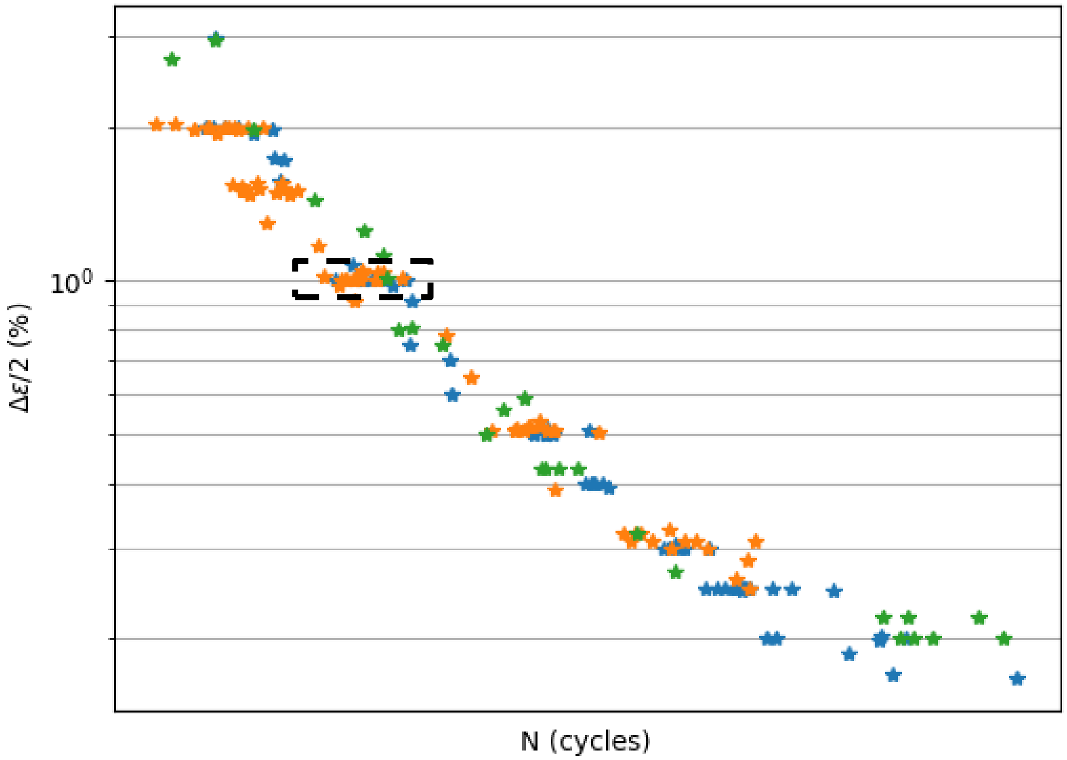
→ How to model the uncertainties linked to fatigue scatter ?

Outline

1. **Fatigue scatter modelling**
2. Variable amplitude loadings
3. Example study case

Fatigue scatter phenomenom

- When performing multiple fatigue tests with the same loadings, there can still be a scatter of the numbers of cycles to failure



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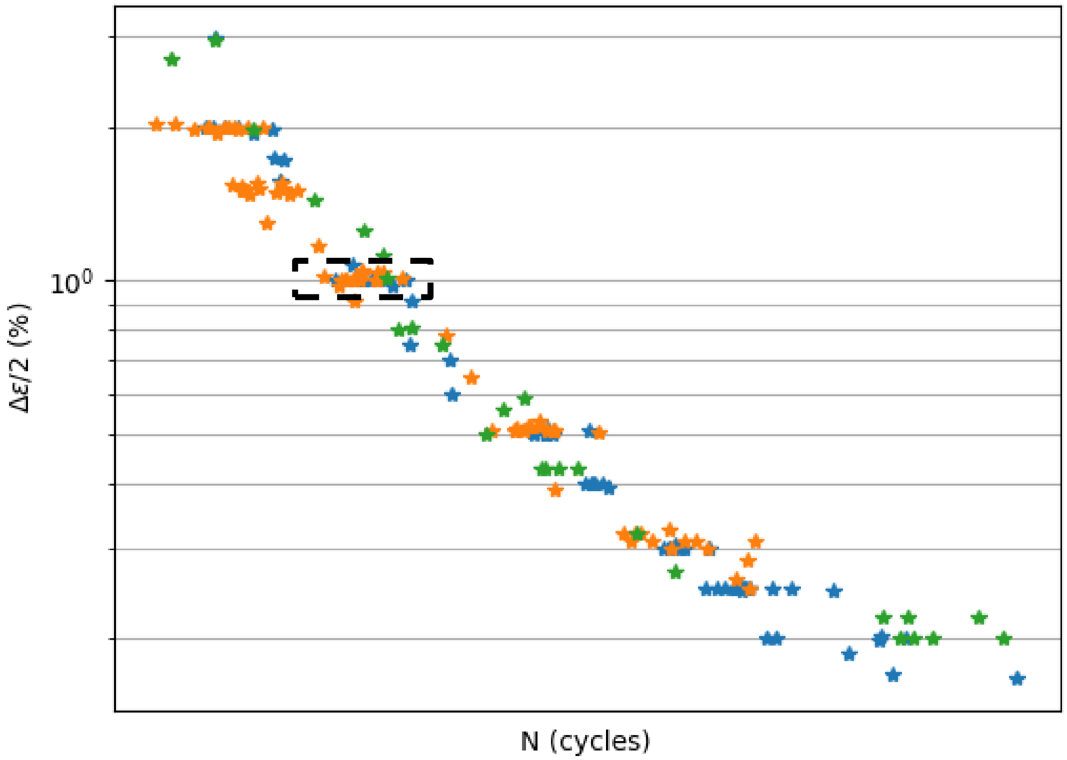
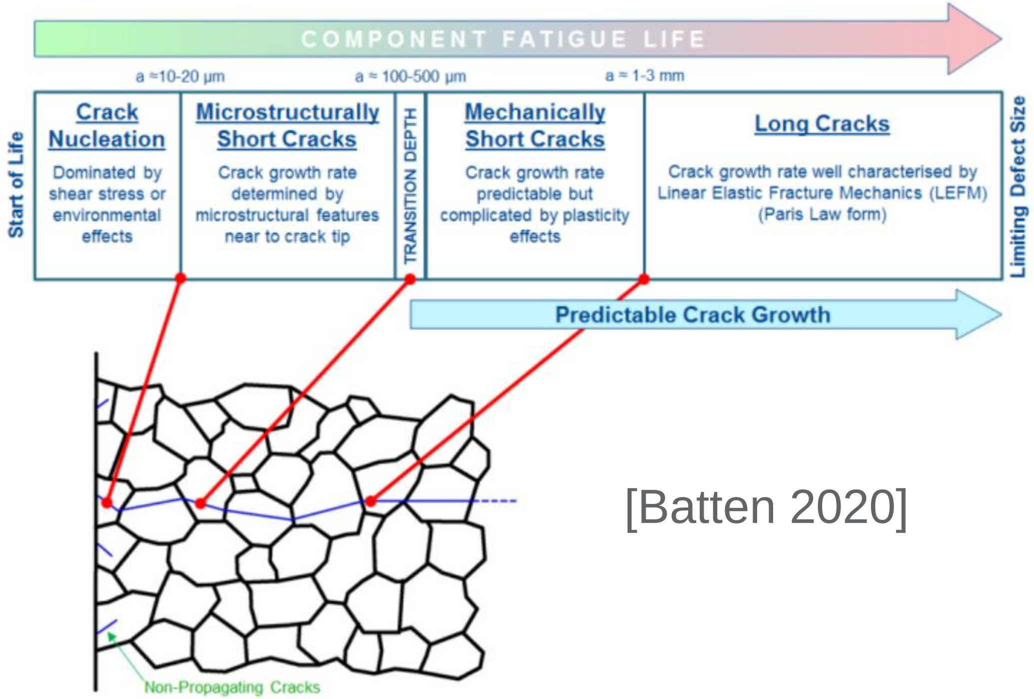


- [Murakami 2021] : Three sources of fatigue life scatter
  - Scatter of the defect sizes
  - Poor alignment of testing machines and specimens
  - **Variability of the microstructure**



[Batten 2020]

Fatigue tests database

- One level LCF tests
  - 316L and 304L austenitic stainless steels
  - Strain controlled, R = -1
  - Controlled temperatures (20°C / 300°C)
  - Controlled surface finish (Polished / Grounded)
  - Controlled testing environment (air / water)

  - **The tests are issued from different laboratories**

Numbers of tests for each experimental conditions :

20°C	Air	Water
Polished	166	/
Grounded	/	/

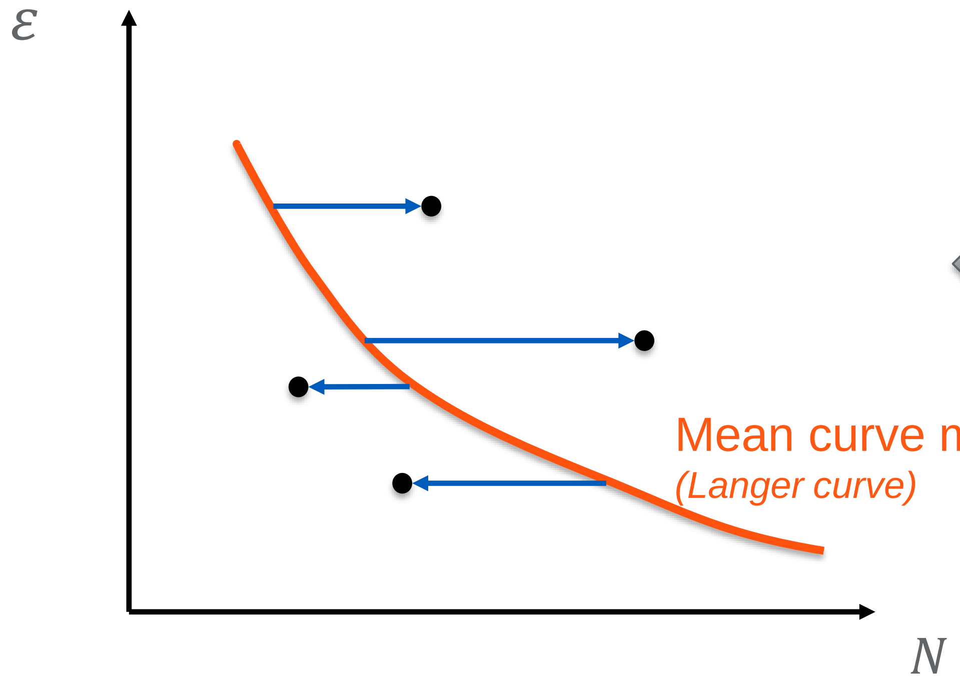
300°C	Air	Water
Polished	84	44
Grounded	27	38

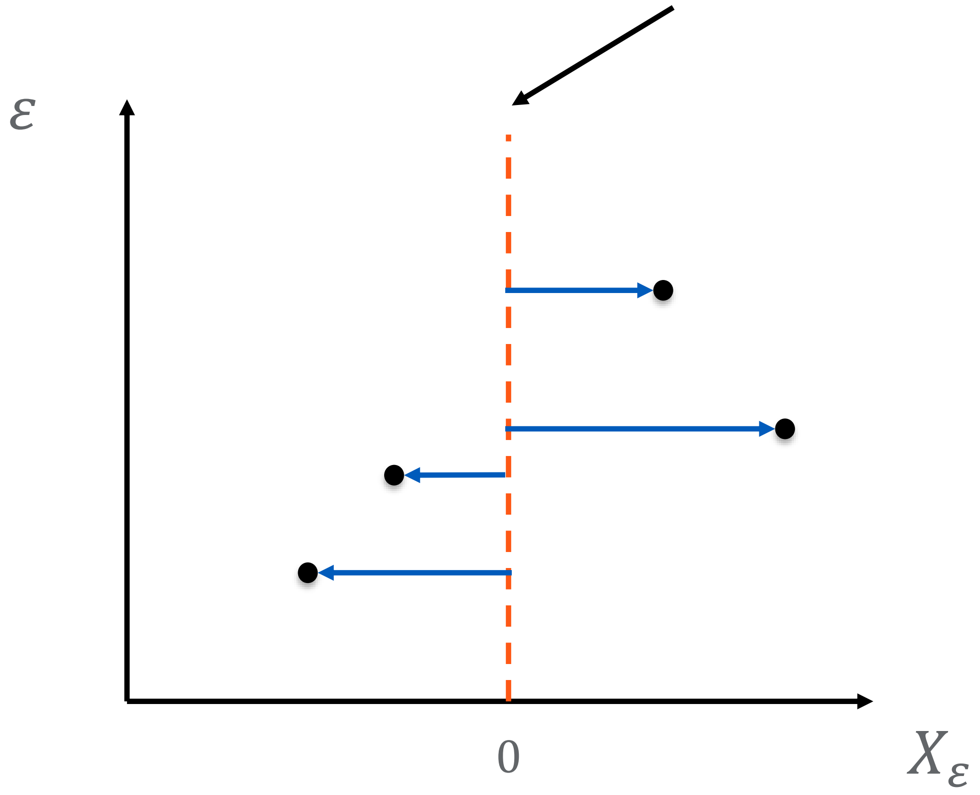
Proposed fatigue scatter model

- Fatigue tests results are the realization of a two dimesionnal random variable (ε, N)

$\rightarrow \ln(N) = \mu(\varepsilon) + X_\varepsilon$

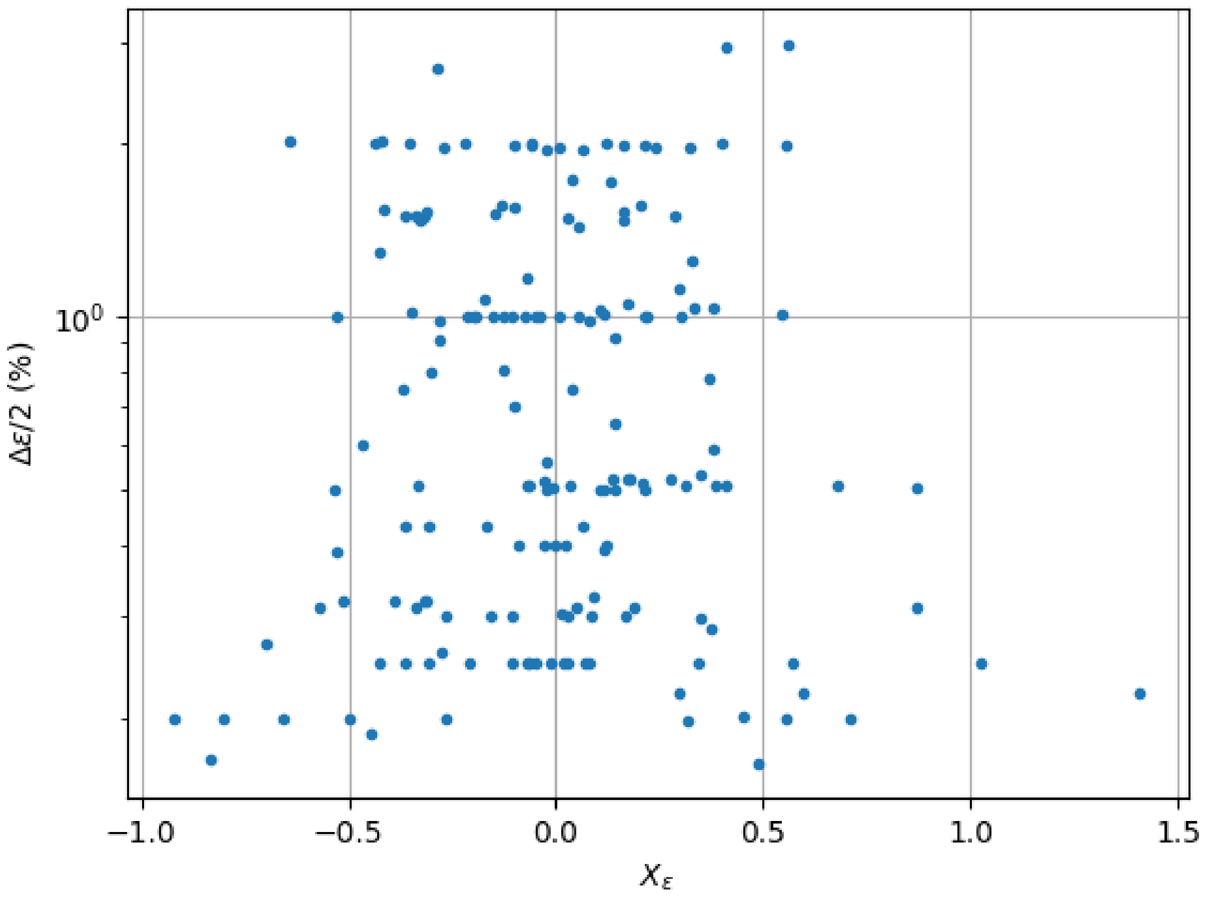
The expected mean value of X_ε is 0





Proposed fatigue scatter model

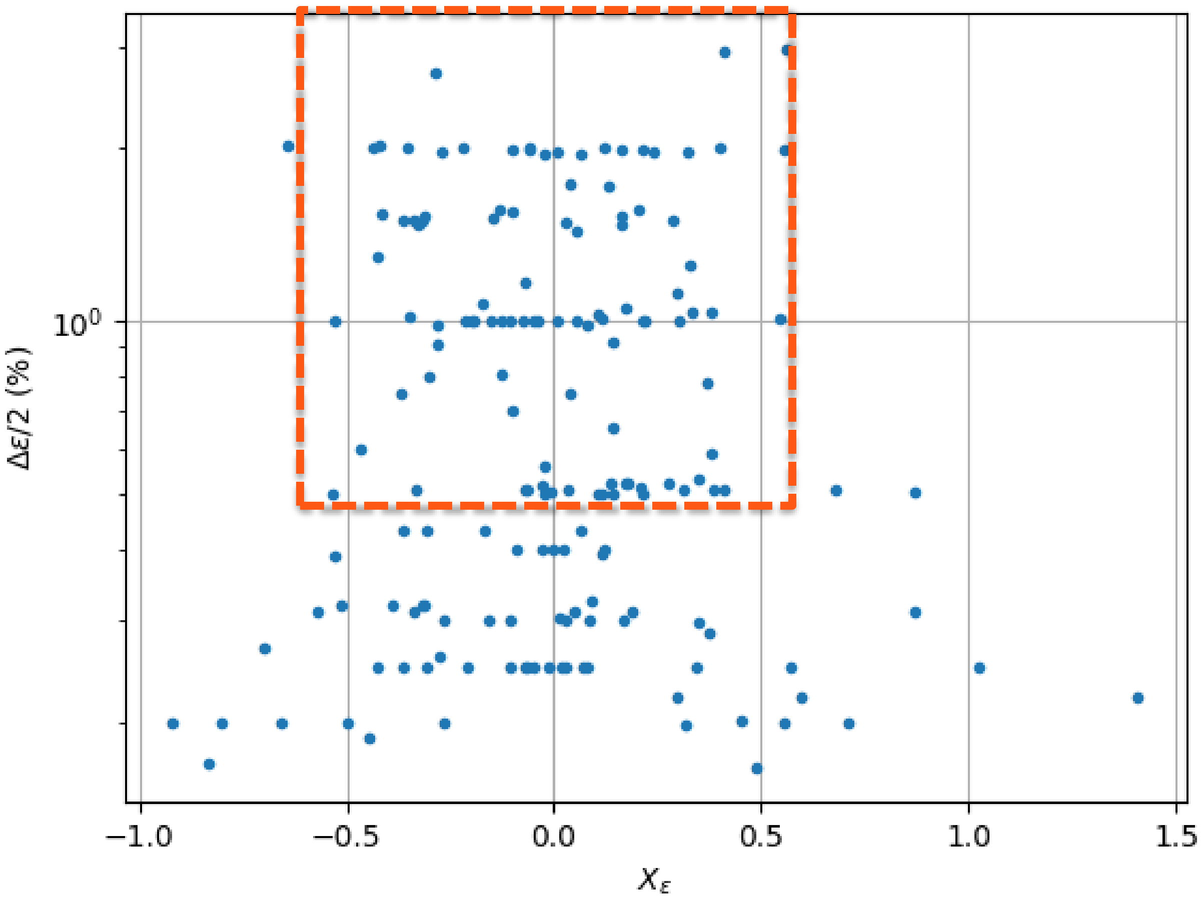
- Observed dependence to the load amplitude of the Langer curve model prediction error X_ε



Proposed fatigue scatter model

- Observed dependence to the load amplitude of the Langer curve model prediction error X_ε

- Choosen model :
  - **For high enough loads, no dependency**
  - For low enough loads, a linear dependency



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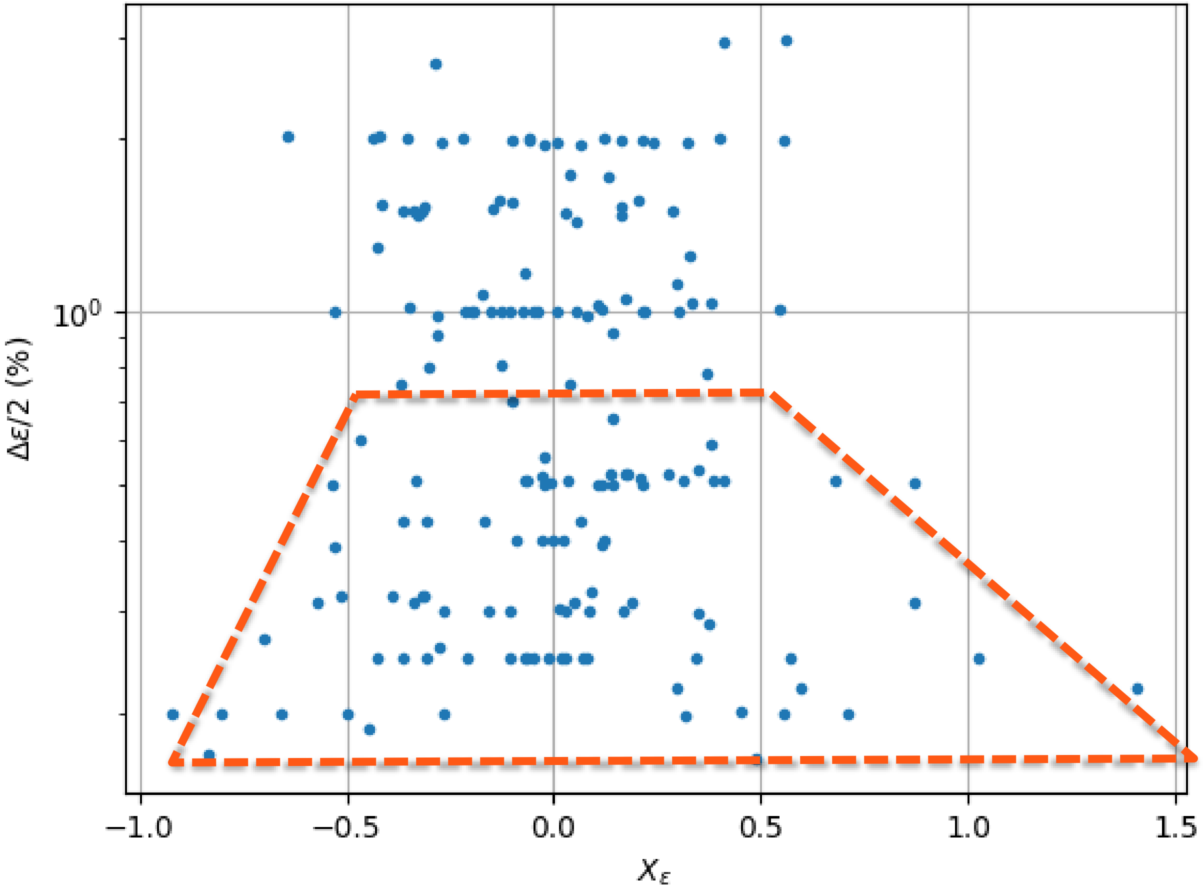


Proposed fatigue scatter model

- **Assumption : The law associated to the scatter of the central tendency model's prediction error only varies with respect to ε by a multiplicative factor $C(\varepsilon)$**

$$\ln(N) = \mu_N(\varepsilon) + C(\varepsilon)X$$

Independent of ε

- Dependancy structure

$$C(\varepsilon) = \max\left(1; 1 + \alpha(\ln(\varepsilon) - \ln(\varepsilon_0))\right)$$

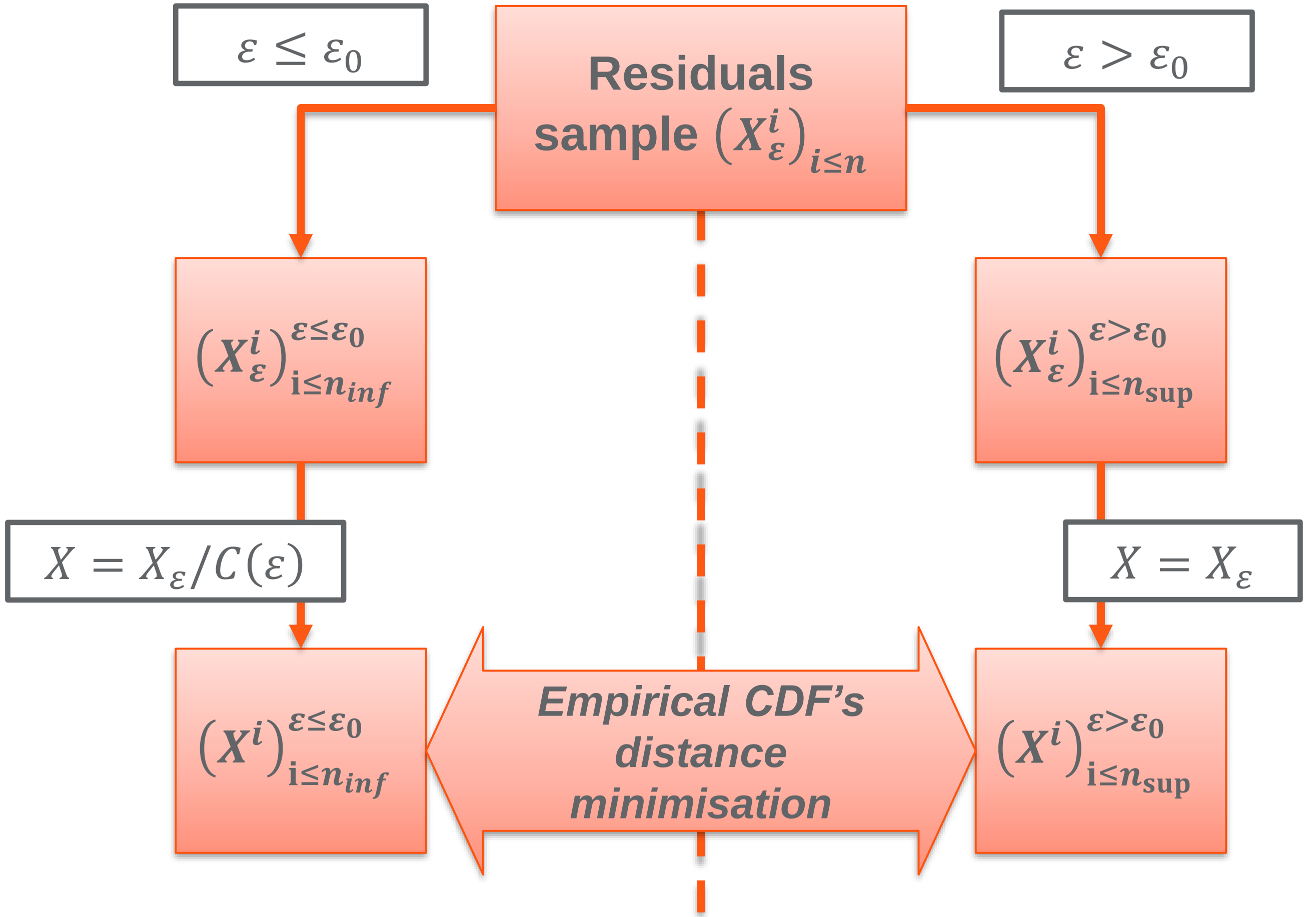
Slope

Behavior threshold

- Identification and validation method based on verifying the above assumption

Parameters identification

- **The law associated to the scatter of the central tendency model's prediction error only varies with respect to ε by a multiplicative factor $C(\varepsilon)$**



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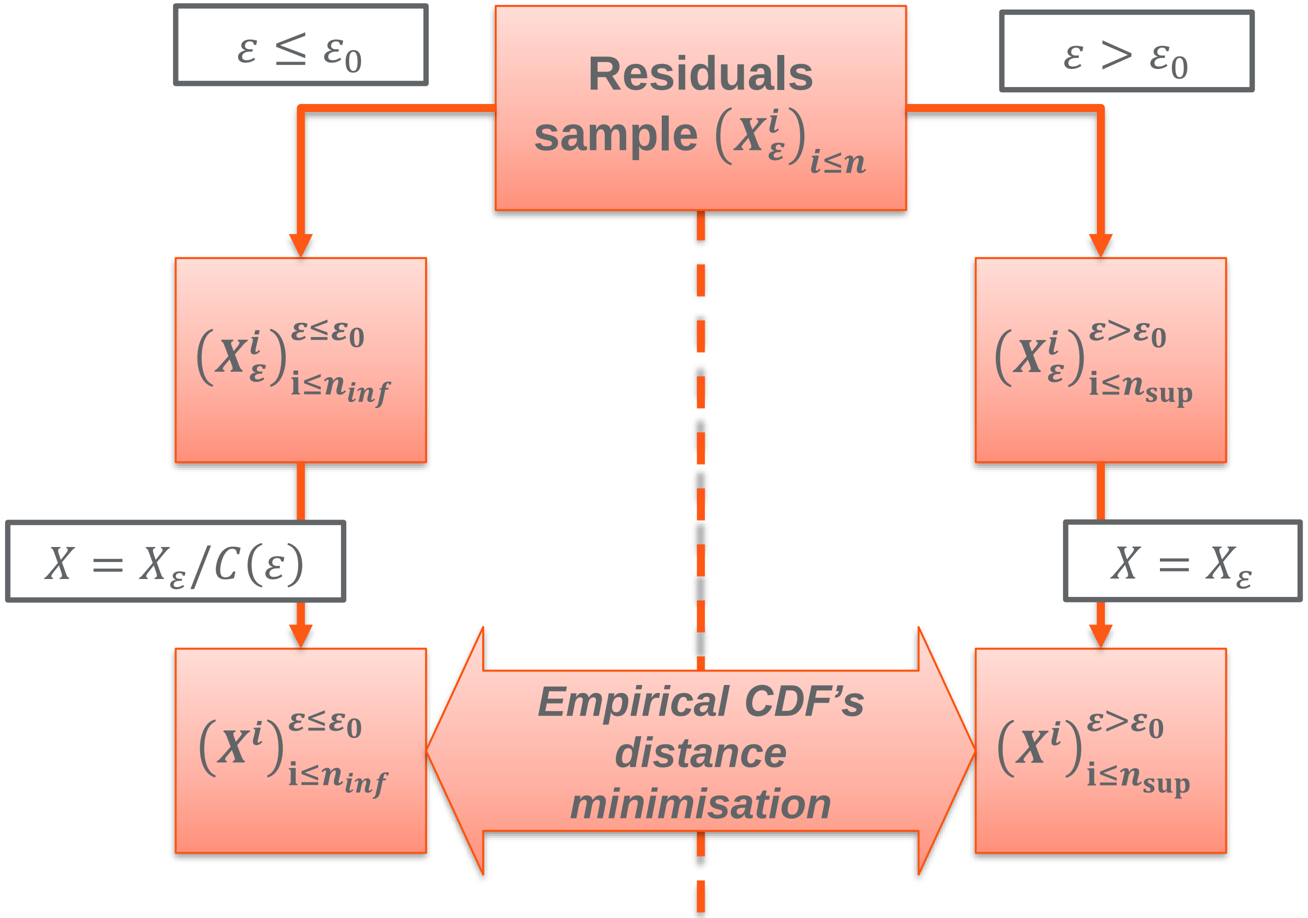
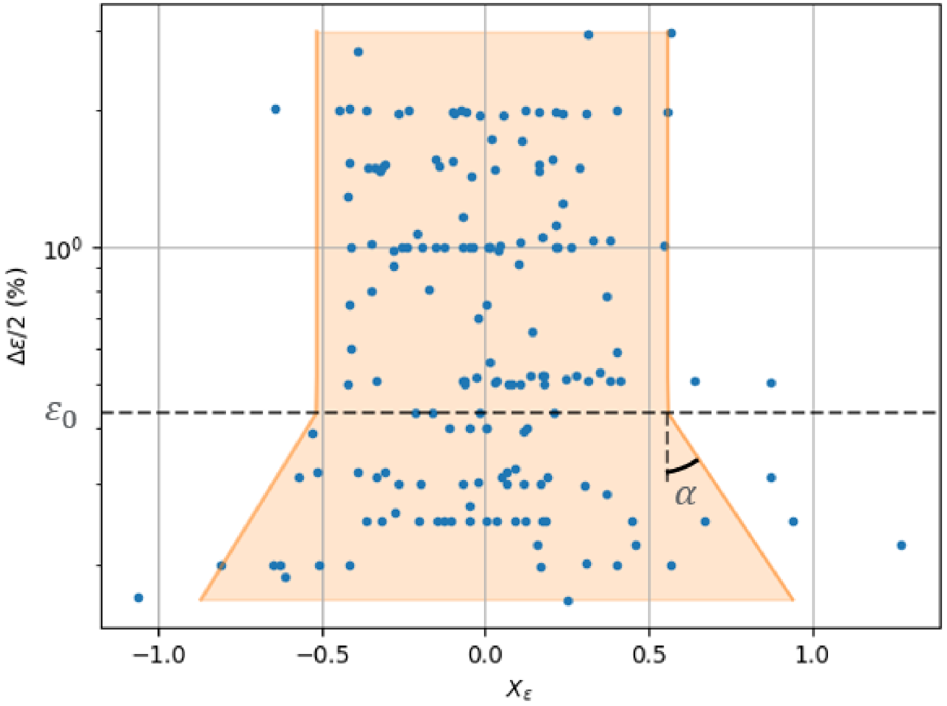
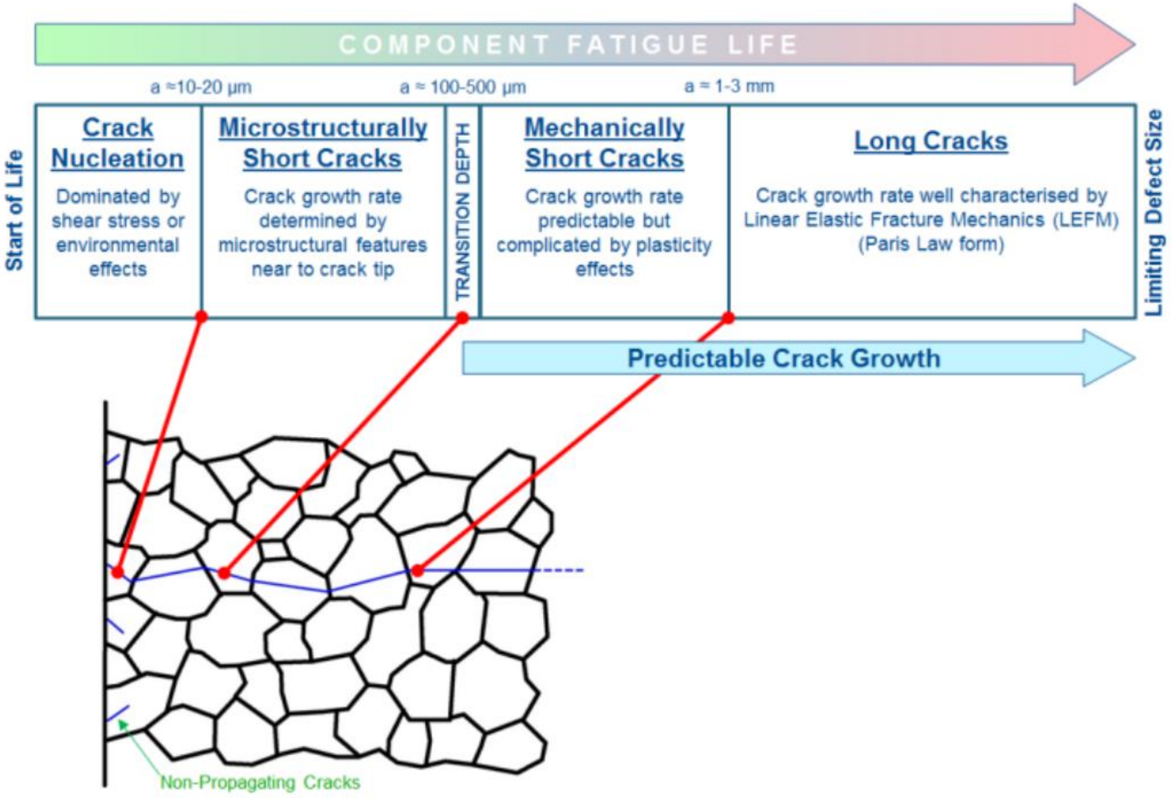


Identification results :



Why the greater scatter for lower loads ?

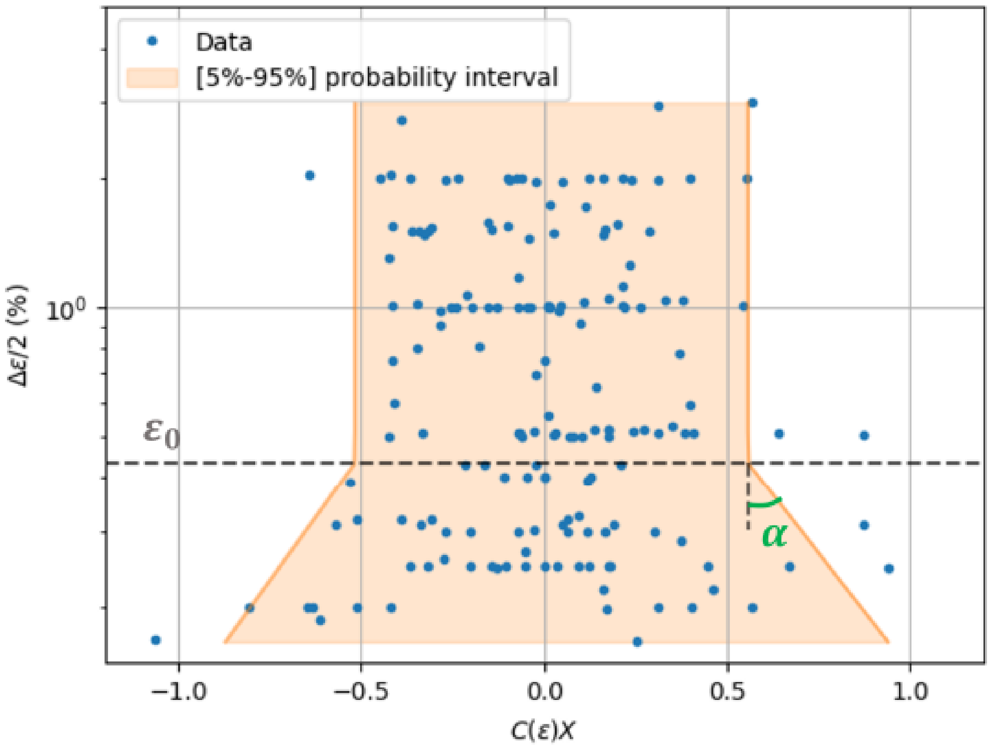


- Sizes of cracks influenced by the microstructure : $20\mu m\ - 500\mu m$
- For higher loads, the crack attains more quickly the « mechanically short » sizes
  - Less time influenced by the microstructure → Less scatter

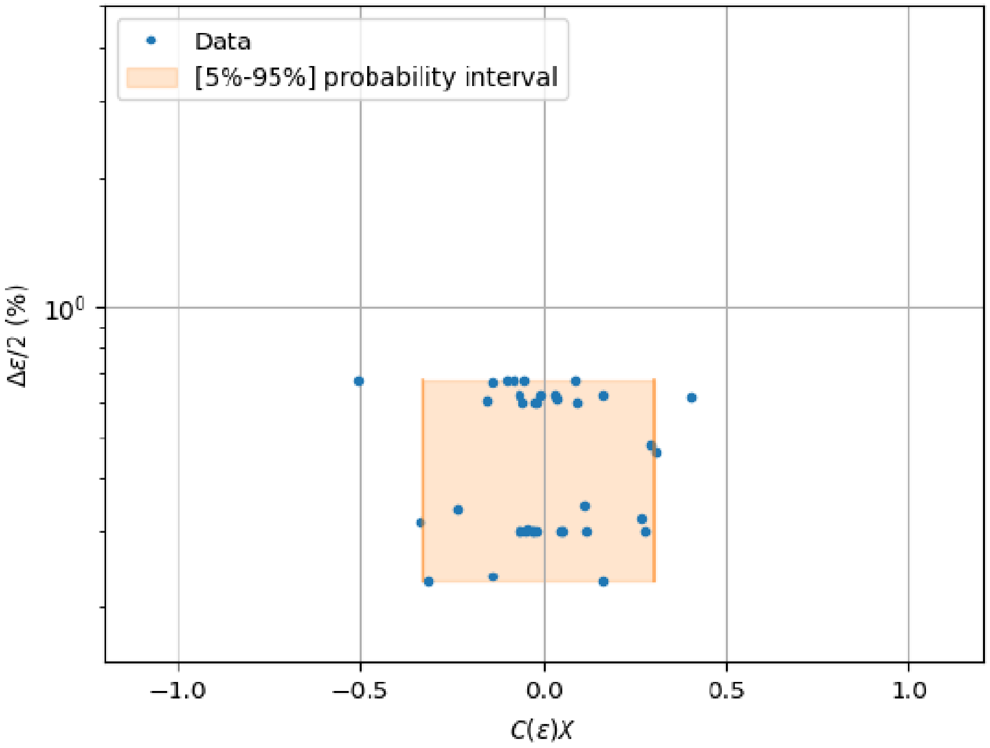
Experimental conditions effect on scatter

- The change for more penalizing experimental conditions reduces fatigue scatter
  - No more effect of the load on fatigue scatter

20°C – air - polished



300°C – water – grounded



Outline

1. Fatigue scatter modelling
2. **Variable amplitude loadings**
3. Example study case

Variable amplitude loadings

- How can we use the resistance scatter model with variable amplitude loadings ?

- Two assumptions [Cartiaux 2023] :

  (A1) The uncertainty of a fatigue life in an experiment **originates solely from the initial state** of the specimen

  (A2) The **fatigue damage** applied to the specimen in each loading cycle **is a deterministic function** of **the current state** of the specimen and of **the load amplitude** of the current fatigue loading

Variable amplitude loadings

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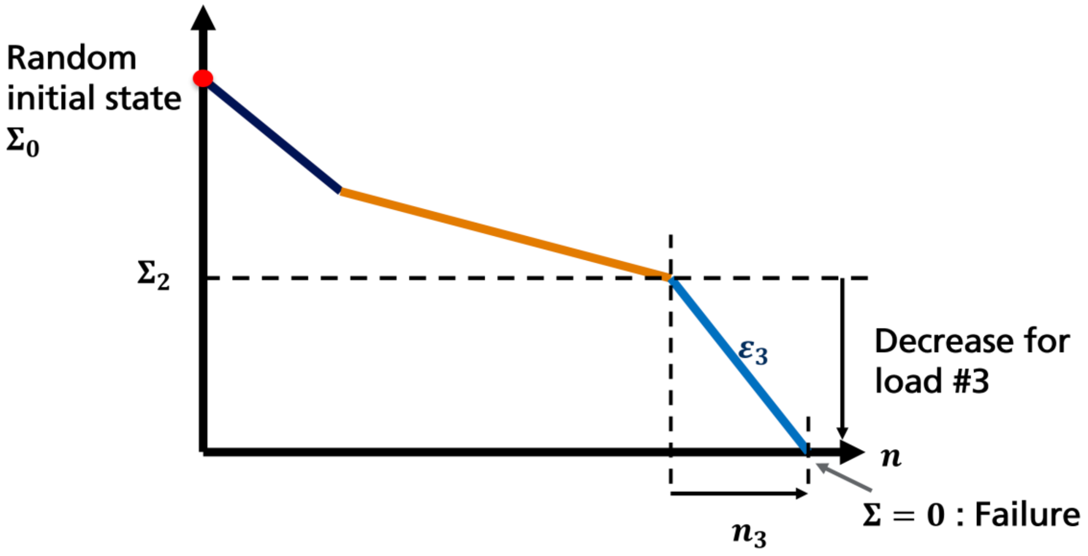


- **The two assumptions allows to define the « health » of a structure**

Variable amplitude loadings

- We can show that :

$$\begin{cases} \text{Assumptions (A1) and (A2)} \\ \ln(N) = \mu(\varepsilon) + C(\varepsilon)X \end{cases}$$

Implies :

$$\begin{cases} \forall \Sigma \in \mathbb{R}^+, \mathbb{P}(\Sigma_0 < \Sigma) = \mathbb{P}(X < \ln(\Sigma)) \\ \Sigma_{n+1} = \left(\Sigma_n^{C(\varepsilon_{n+1})} - U_{n+1}\right)^{1/C(\varepsilon_{n+1})} \end{cases}$$

The law of Σ_0

The law of $\Delta\Sigma$

Health decrease law

$$\Sigma_{n+1} = \left(\Sigma_n^{C(\varepsilon_{n+1})} - U_{n+1}\right)^{1/C(\varepsilon_{n+1})}$$

Load #i+1 amplitude

Health before load #i+1

Usage factor $\frac{n_{i+1}}{N_{i+1}}$ computed with the mean fatigue curve

- This is a non linear cumulative damage law !

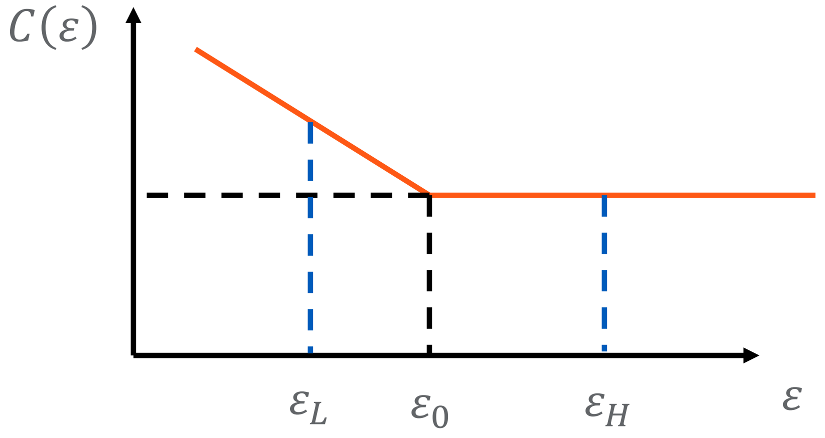
Non linear properties

The model represents non linear cumulative effects :

- For two level H and L loads ($\varepsilon_H > \varepsilon_L$)

  - If $C(\varepsilon_H) < C(\varepsilon_L)$, $\Delta\Sigma_{H\to L} > \Delta\Sigma_{L\to H}$

  → H-L loads are more damaging than L-H loads



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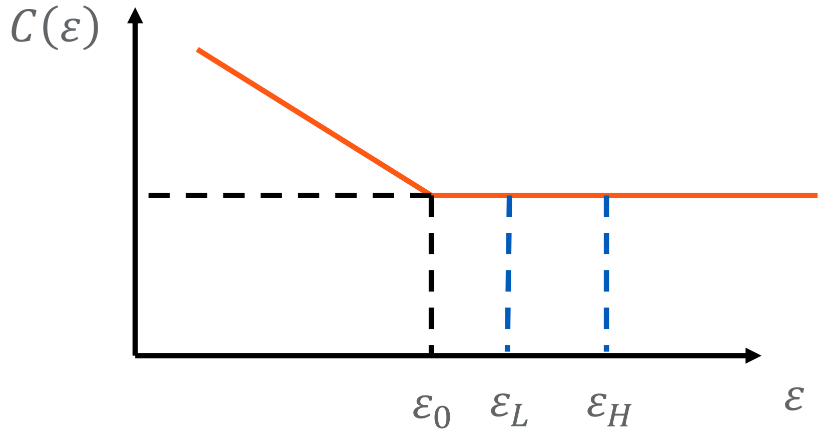
  - If $C(\varepsilon_H) = C(\varepsilon_L), \Delta\Sigma_{H\to L} = \Delta\Sigma_{L\to H}$

  → For high enough loads, the damage law becomes linear [Cartiaux 2023]



How to link fatigue life scatter and the non linear cumulative damage model ?

- The non linear cumulative model is obtained solely through :
  - The fatigue scatter model (Non linearity is due to the scatter dependency to the load amplitude)
  - Assumption A1 : The uncertainty of a fatigue life in an experiment **originates solely from the initial state** of the specimen
  - Assumption A2 : The **fatigue damage** applied to the specimen in each loading cycle **is a deterministic function** of **the current state** of the specimen and of **the load amplitude** of the current fatigue loading

- This means that the crack growth effects explaining fatigue scatter can also explain the non linear cumulative effects

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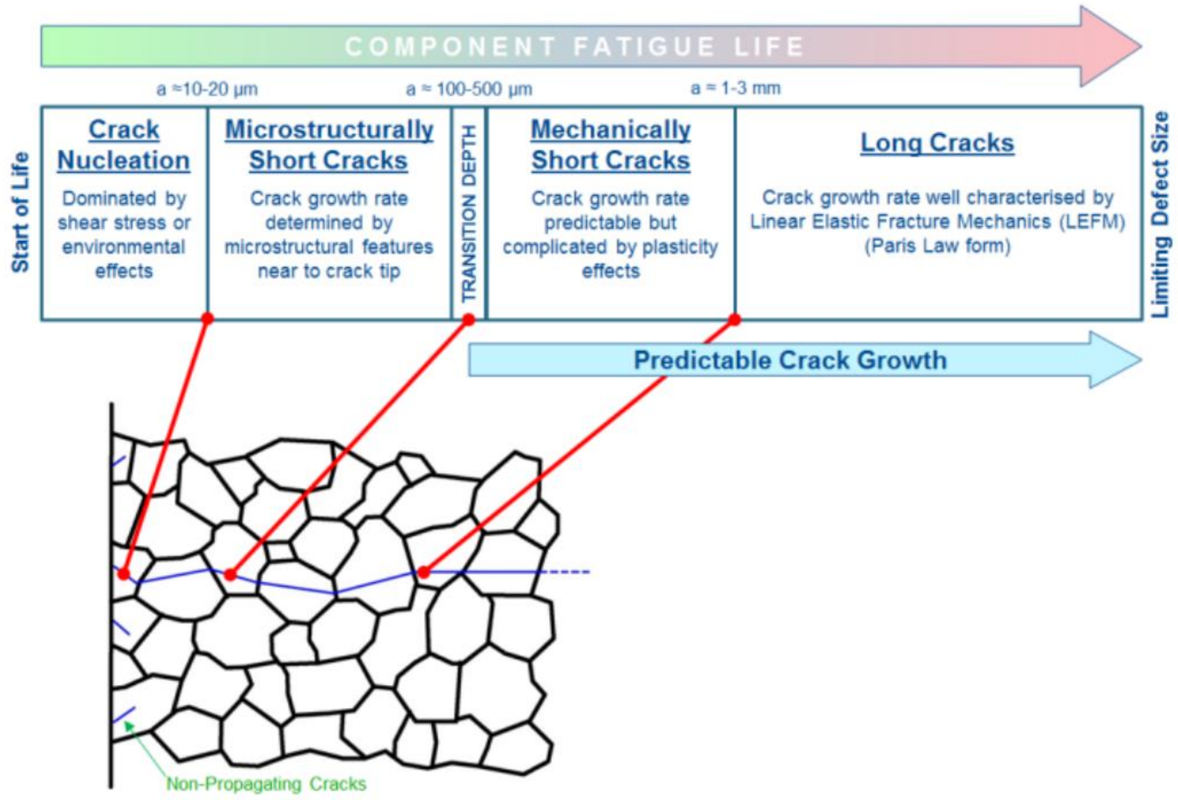
- This means that the crack growth effects explaining fatigue scatter can also explain the non linear cumulative effects



- ➢ Higher loads means the critical crack attains the "mechanically short cracks" state more quickly
- ➢ When a crack has not reached the "mechanically short cracks" state, its growth is affected by the microstructure
- ➢ The microstructure effect can slow down considerably the crack growth, especially for lower loads

Limits of the model regarding the assumption A2

- Assumption A2 : The **fatigue damage** applied to the specimen in each loading cycle **is a deterministic function** of **the current state** of the specimen and of **the load amplitude** of the current fatigue loading **and of the previous load amplitudes**
  - Example : Crack overload effect

- This assumption change is easily integrable into our model :

$$\Sigma_{n+1} = \left(\Sigma_n^{C(\varepsilon_{n+1})} - U_{n+1} - \underbrace{f\left(\varepsilon_{n+1}, (\varepsilon_i)_{i\leq n}\right)}\right)^{1/C(\varepsilon_{n+1})}$$

Generic function such that if ε_n is constant, $f = 0$

Effect of penalising conditions on non linearity

- Growth of the fatigue scatter with load amplitude → Non linear cumulative damage
- Penalising experimental conditions (300°C – water – grounded) → No dependency towards the load amplitude

→ **The non linear cumulative damage effects measured on laboratory conditions should be at least reduced when considering the actual industrial structure**

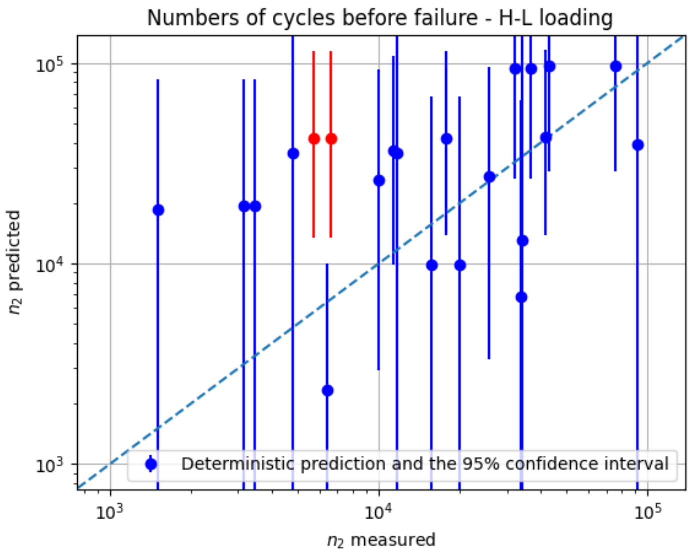
Model validation

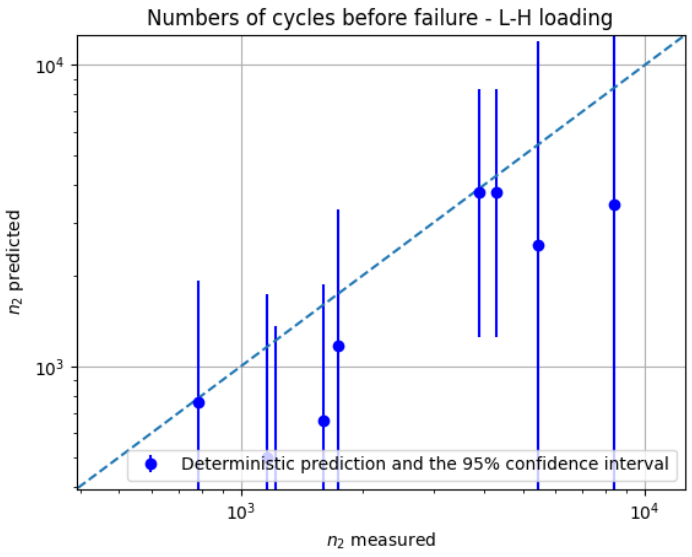
- Validation with two level fatigue tests
- The model is used to predict the numbers of cycles to failure after the second load :

$$n_2(\Sigma_0) = N(\varepsilon_2)\left(\Sigma_0^{C(\varepsilon_1)} - U_1\right)^{\frac{C(\varepsilon_2)}{C(\varepsilon_1)}}$$

- If the model is correct, n_2 has a 95% chance to be included in $\mathrm{I}_{95\%} = [n_2(q_{2.5\%})\,; n_2(q_{97.5\%})]$

  → **Statistical hypothesis test**





29/31 points are included in $I_{95\%}$, the model cannot be rejected with a confidence of 95% ($p > 0.05$)

How to use the health model with random loadings ?

- Consider a load case with n loadings : $\varepsilon_1, \dots, \varepsilon_n$
- What is the initial health value Σ_0^r that leads to failure after load n ?

$$\Sigma_n^r = 0$$

$$\Sigma_{n-1}^r = \left({\Sigma_n^r}^{C(\varepsilon_n)} + U_n\right)^{1/C(\varepsilon_n)}$$

$$\dots$$

$$\Sigma_0^r = \left({\Sigma_1^r}^{C(\varepsilon_1)} + U_1\right)^{1/C(\varepsilon_1)} = f(\varepsilon_1, \dots, \varepsilon_n)$$

- Failure happens if the initial health is lower than Σ_0^r, the failure event becomes :

$$\Sigma_0 < \Sigma_0^r(\varepsilon_1, \dots, \varepsilon_n)$$

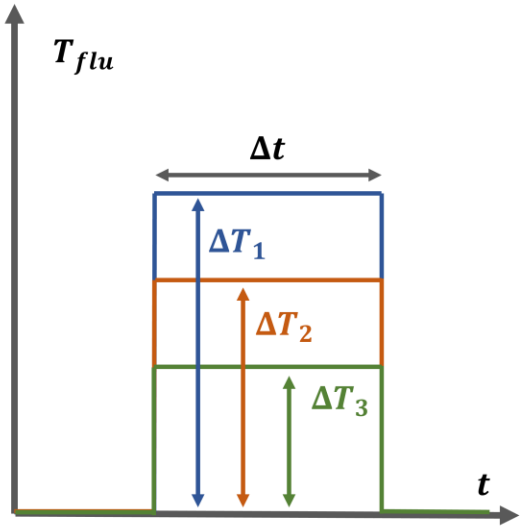
Independant structural resistance term

Structural load term

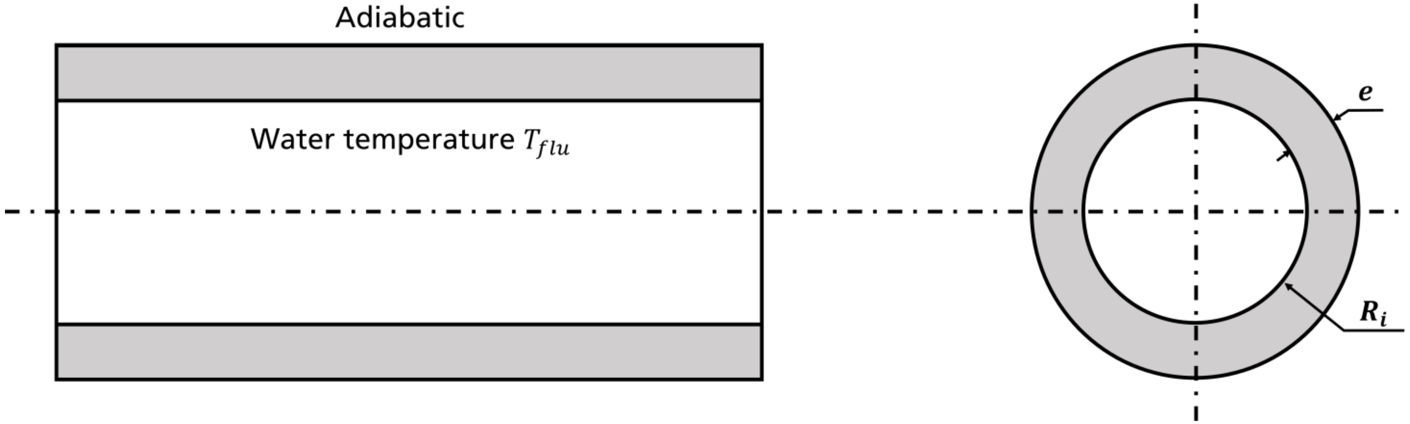
Outline

1. Fatigue scatter modelling
2. Variable amplitude loadings
3. **Example study case**

Study case definition



- Simplified pipe [Guede 2005] under thermal shocks case



Uncertain parameters :

- Fatigue load $(\Delta T_i, \Delta t)$
- Thermomecanical model $(E, h, \alpha, \ldots)$
- Pipe geometry (R_i, e)

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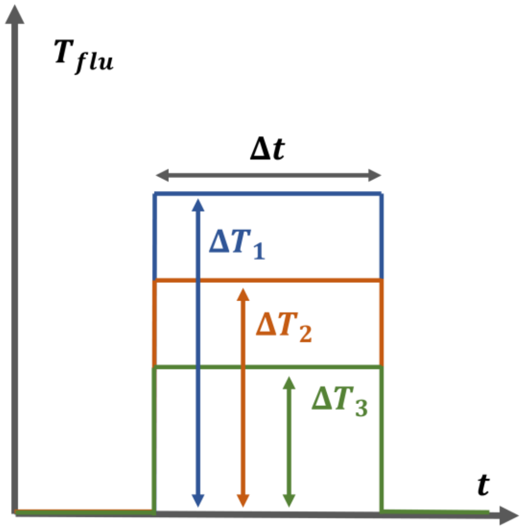

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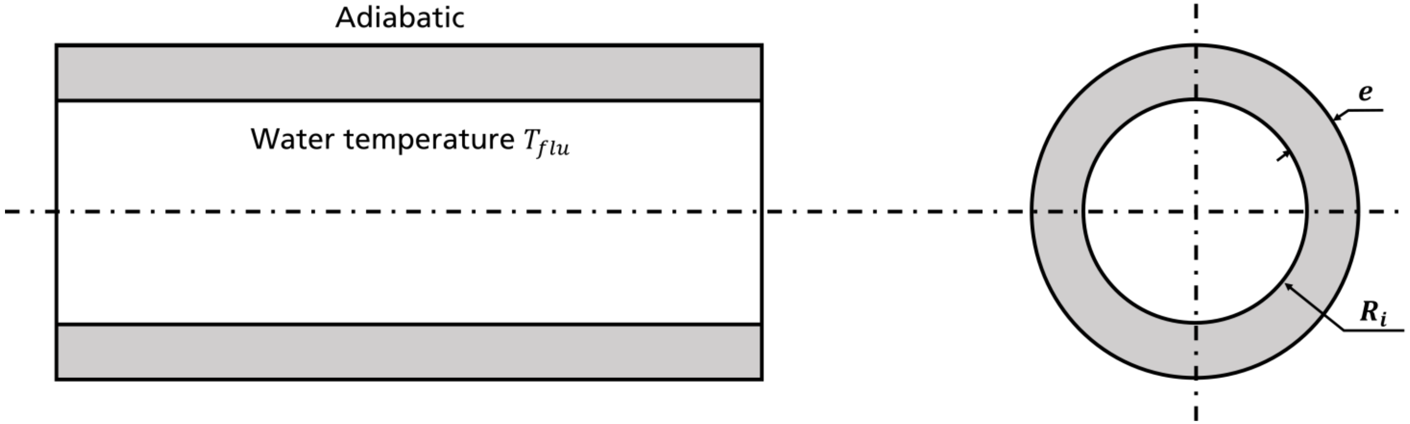

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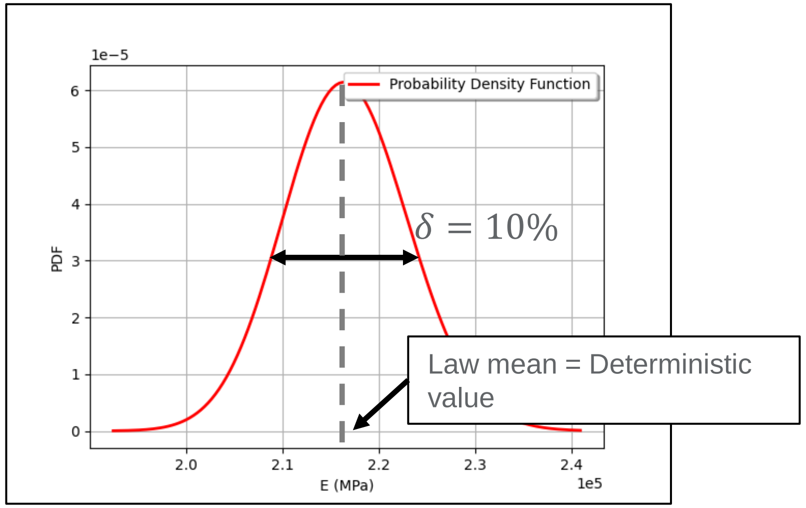
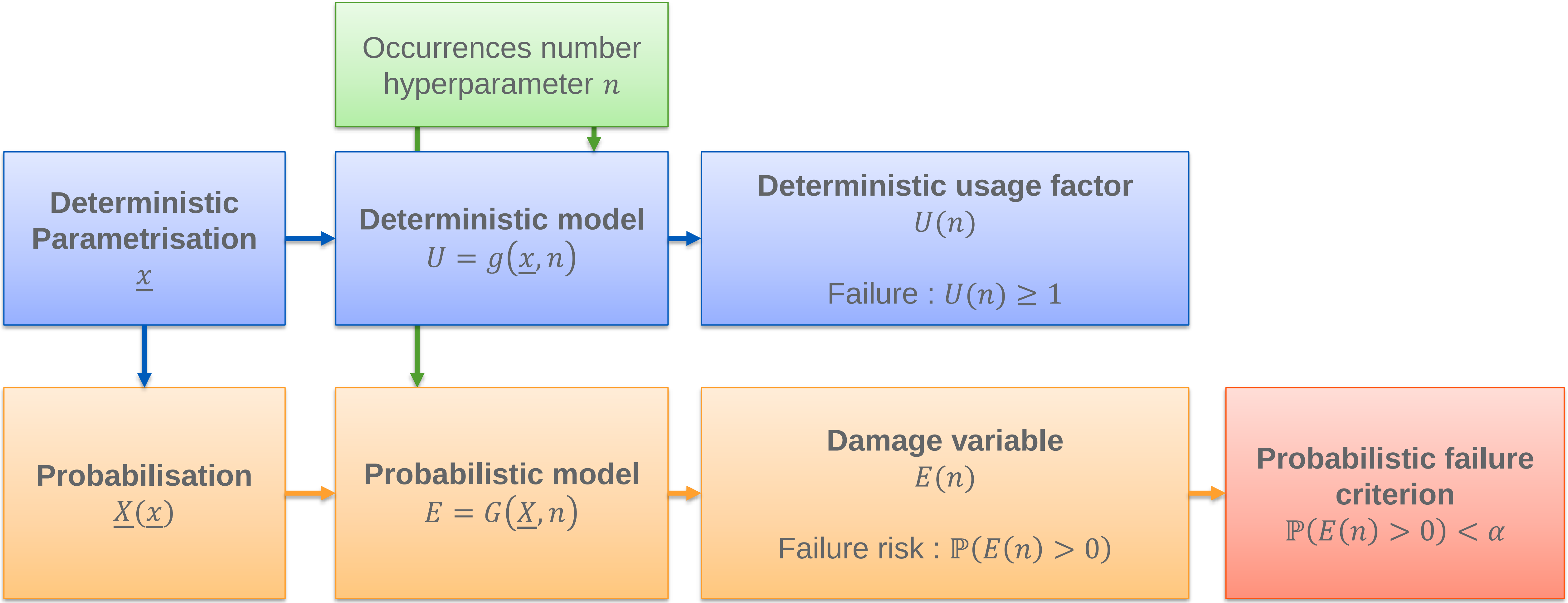


Critical usage factor – Definition (1/2)



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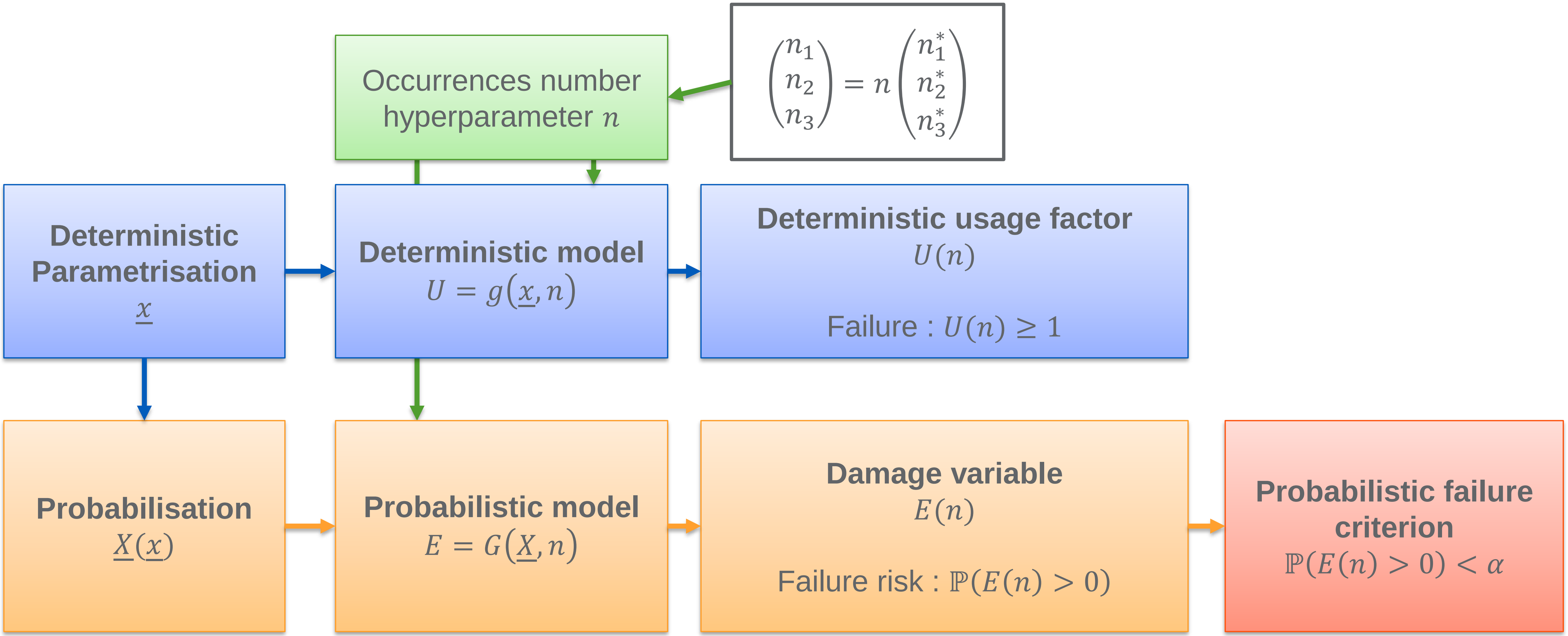
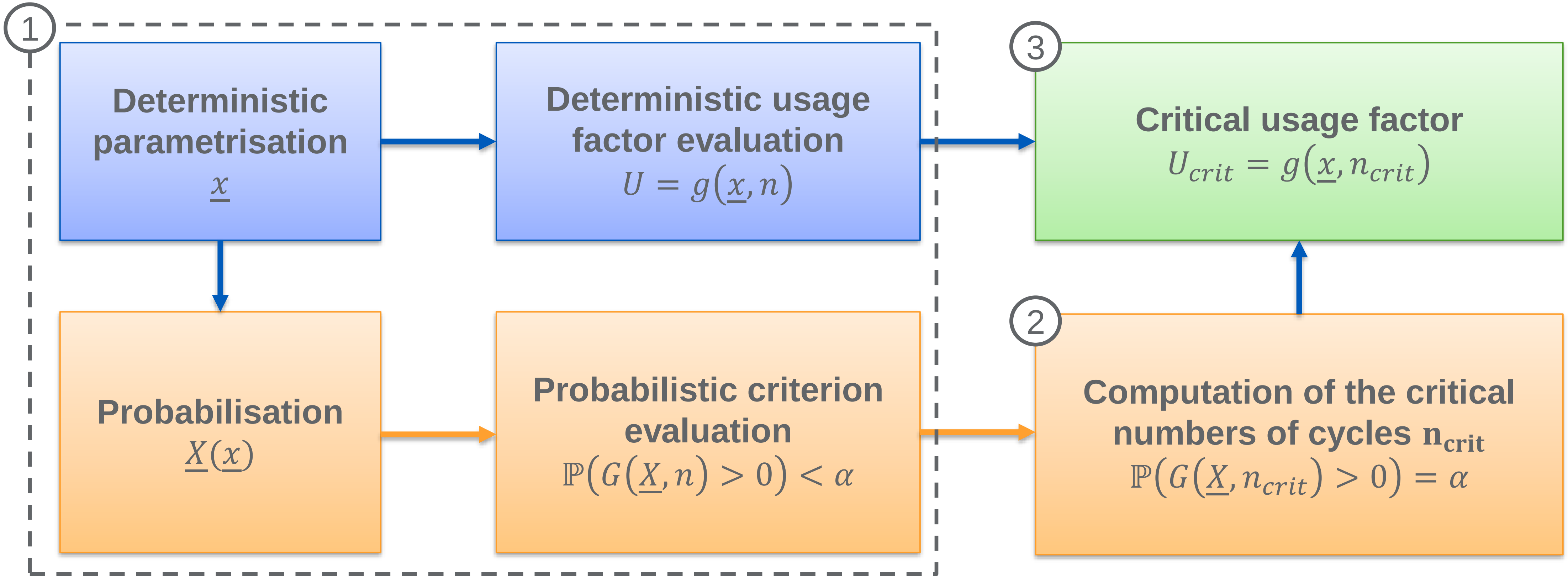


Critical usage factor – Definition (2/2)



Failure, according to the probabilistic model, happens when $\boldsymbol{n \geq n_{crit}}$, i.e. $\boldsymbol{U \geq U_{crit}}$

Effect of the fatigue life scatter on the probabilistic

→ The deterministic criterion does not take into account the load sequence, but the probabilistic criterion does

- H-L sequence : $U_{crit} = 0{,}99$
- L-H sequence : $U_{crit} = 2{,}10$
- Random sequence : $U_{crit} = 1{,}35$

➢ The choice of the load sequence is not required in the RCC-M, and the potential sequence effects are taken into account by penalizing the fatigue SN curve

  ➢ This method is too conservative !

Conclusion

- Generic scatter model definition and identification
  - No assumptions on the scatter distribution X and on the mean fatigue curve $\mu(\varepsilon)$
  - Ajustable load dependency description $C(\varepsilon)$
  - Can be used with any fatigue severity (Strain amplitude, stress amplitude, energetic criterion, …)

- Variable amplitude framework
  - Based on two simple assumptions → The assumptions can be easily extended
  - Adapted to the scatter model definition → Predicts non linear cumulative effects

- Standard load-resistance failure event redifinition

$$\Sigma_0 < \Sigma_0^r(\varepsilon_1, \dots, \varepsilon_n)$$